A GRASP metaheuristic for a districting problem arising in urban distribution

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Abstract

This paper presents a simple greedy randomized adaptive search procedure (GRASP) embedded in a decision support system for the design of delivery districts in urban distribution. The proposed GRASP aims at balancing the workload of delivery routes by considering both the travel time within customers and the service time at the customers. To compute travel times we use a continuous approximation of the length of a travelling salesman tour for each district. Computational experiments, with the data of a major Colombian company serving hundreds of stores a day, reveal a potential reduction of the workload imbalance of their routes by more than 50%.

1 Introduction

Urban distribution/City logistics is about finding efficient and effective ways to transport goods in urban areas while taking into account the negative effects on congestion, safety, and environment.[1]. Urban distribution in emerging markets faces additional challenges due to the important market share of the traditional channel. This channel comprises family owned business that range from mini-stores (15-40 sq. ft. of store surface) to nano-stores (less than 15 sq. ft. of store surface) and street carts [2]. While planning the distribution process to these stores theirs suppliers (dairy, food, and soft-drink producers among others) have to partition the stores into territories or districts in order to assign their marketing (sellers) and delivery (trucks) resources efficiently [3]. In this work we describe the metaheuristic embedded in a decision support system employed by a major Colombian food company for the districting of their customers.

1.1 The districting problem

The company under study serves hundreds of customers (modern retail stores, mini- and nano-stores) a day in all major Colombian cities. In their tactical planning process, they solve a districting problem (DP) that aims to have an even workload for the crews assigned to their delivery territories. The DP they solve can be defined as follows. Given a set of customers $\mathcal{C}$, each one with a service time $t_i, (i \in \mathcal{C})$, and a set of districts $\mathcal{K}$,. DP assigns each customer to one and only one delivery district, by partitioning the set of customers into $|\mathcal{K}|$ disjoint subsets. Binary variables $x_{ik}, i \in \mathcal{C}, k \in \mathcal{K}$ define such partitioning ($x_{ik} = 1$,.if customer i is assigned to district k and $x_{ik} = 0$, otherwise). Using these variables, the workload for a given district k is defined by equation (1). The first term of this equation takes into account the total service time whereas the second term ($tt(k)$), corresponds to the approximation of the total travel time of the route serving customers on district k.

$$w_k = \sum_{i \in \mathcal{C}} t_i x_{ik} + tt(k), \forall k \in \mathcal{K} \tag{1}$$

Following, the approach proposed in [4], we use the classical Beardwood–Halton–Hammersley formula to calculate the expected travel time of the tour serving the customers assigned to a district [5]. Then, given the number of customer assigned to the district (n_k) and the area of the district (A_k), equation (2) defines the value of $tt(k)$. In this expression, β is a constant that we calculate empirically using the data from the company.

$$tt(k) \approx \beta\sqrt{A_k n_k}, \tag{2}$$

Finally, the proposed DP seeks to minimize a composite objective function with two terms: (i) the average workload $\left(\overline{w} = \frac{\sum_{k\in\mathcal{K}} w_k}{|\mathcal{K}|}\right)$, and the average absolute deviation with respect to $\overline{w}$ $\left(\overline{\Delta} = \frac{\sum_{k\in\mathcal{K}} |w_k - \overline{w})|}{|\mathcal{K}|}\right)$.

2 A GRASP for districting stores in urban distribution

Districting problems have been studied in different applications ranging from political districting, to sales territory design [6]. To solve these problems the preferred solution methods are metaheuristics, including adaptive large neighborhood search [4], tabu search [7], greedy randomized adaptive search procedures (GRASP) [8], among others. In view of that, to solve the DP we implemented a simple GRASP [9] comprising a greedy randomized construction phase (`BuildDistricts`) and a local search phase (`ImproveDistricts`) that repeat over `T` iterations. These two phases aim at generating compact districts with a balanced workload between delivery districts.

2.1 Greedy randomized construction

The construction phase seeks to create compact districts taking into account only the service time of the customers assigned to them. To do so, procedure `BuildDistricts` creates districts sequentially using a simple nearest neighbor heuristic. To build a district, it first selects a random seed customer and then adds the unassigned customer that is closer to the centroid of the district. The insertion of customers in the district iterates until its cumulative service time exceeds the ideal value $\overline{t} = \frac{\sum_{i\in\mathcal{C}} t_i}{|\mathcal{K}|}$. Note that each time a customer is assigned to a district, the procedure `BuildDistricts` updates the set of unassigned customers, the centroid of the district and its cumulative service time. Once $\overline{t}$ is surpassed, this procedure closes the current district and selects randomly an unassigned seed customer to build a new district. The construction phase stops once $|\mathcal{K}|$ districts has been created. In the last district, the procedure ignores the limit $\overline{t}$ and assigns all remaining unassigned customers.

2.2 Local search

In the local search phase, we use equation (2) to update the workload of each district by considering the second term of equation (1) (i.e., its travel time). Then, in a greedy fashion procedure `ImproveDistricts` selects the district k' with the maximum deviation to $\overline{w}$ and tries to reduce the difference. If $w_{k'} < \overline{w}$, the procedure select the customer c' not assigned to district k' that is closer to it centroid and reassigns this customer to it. On the other hand, if $w_{k'} > \overline{w}$, the procedure takes the customer c' assigned to $w_{k'}$ that is farther to its centroid and reassigns it to another district. When reassigning c' the procedure selects the closer district (i.e., the district different from k' with the closest centroid). In the local search phase we implement a nonmonotone GRASP [10], therefore `ImproveDistricts` stops after `u` non-improving movements.

3 Case study results

The proposed GRASP has been implemented using Visual Basic for Applications (VBA) and is embedded in a decision support system with Excel as main user interface. To calculate the relevant geographic information of the customers and to display the results we use their Microsoft Power Maps extension. A run of the proposed GRASP takes about five minutes to obtain a solution in a standard desktop computer. To evaluate the effectiveness of our GRASP, we compared the current districting of the company with respect to the one obtained with the proposed approach in a small Colombian city with 800 customers (stores). The current districting of the company is mainly obtained using the Network Analyst toolbox for ArcGIS to obtain compact districts but ignoring the balancing of the workloads. Table 1 summarizes the results of this comparison. The table presents the values in minutes for $\overline{w}$ and $\overline{\Delta}$. To illustrate the improvement in the work balance it also presents the maximum deviations above (Δ^+) and below (Δ^-) $\overline{w}$, and the range ($R_w = \Delta^+ + \Delta^-$) of w_k over all districts. Using a GRASP for the districting of the company reduced the average absolute deviation of the workload by 55.5%, from 63 minutes to 28 minutes, a similar improvement is achieved (54.8%) if the imbalance is measured using R_w (i.e., the range of the workloads). Moreover, Figure 1 illustrates the districts obtained with the proposed approach showing that this solution has a better workload balance without losing the compactness of the districts. Additional results in a test instance from a larger city with 3000 customers will be presented at the conference.

Performance measure (minutes)	$\overline{w}$	$\overline{\Delta}$	Δ^{+}	Δ^{-}	R_w
Company	496	63	67	57	124
GRASP	491	28	23	33	56
Improvement	1.0%	55.5%	65.7%	42.1%	54.8%

Table 1: Comparison of the GRASP results with the districting of the company in a small city

Figure 1. Delivery districts for a small city with 800 customers

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