GRASP/VND with Path Relinking for the Truck and Trailer Routing Problem

Juan G. Villegas[1,2], Christian Prins[1], Caroline Prodhon[1], Andrés L. Medaglia[2] and Nubia Velasco[2]

[1] Laboratoire d'Optimisation des Systèmes Industriels, Institut Charles Delaunay, Université de Technologie de Troyes BP 2060, 10010 Troyes Cedex, France

[2] Centro para la Optimización y Probabilidad Aplicada, Departamento de Ingeniería Industrial, Universidad de los Andes. A.A. 4976, Bogotá D.C., Colombia

Email: jg.villegas64@uniandes.edu.co, juan_guillermo.villegas@utt.fr

1. Problem definition and literature review

The Truck and Trailer Routing Problem (TTRP) is an extension of the well known vehicle routing problem. In the TTRP an heterogeneous fleet composed of m_t trucks and m_r trailers ($m_r < m_t$) is used to serve a set of customers $N = \{1,\ldots,n\}$ from a central depot, denoted with 0 . Each customer $i \in N$ has a demand q_i ; the capacities of the trucks and the trailers are Q_t and Q_r , respectively; and the distance c_{ij} between any two points $i, j \in N \cup \{0\}$ is known. The existence of accessibility constraints at some customers creates a partition of N into two subsets: the subset of *truck customers* N_t accessible only by truck; and the subset of *vehicle customers* N_v accessible either by truck or by a complete vehicle (i.e., a truck pulling a trailer). Due to the heterogeneity of the fleet and the accessibility constraints, a solution of the TTRP may have three types of routes: *pure truck routes* performed by a truck visiting customers in N_v and N_t ; *pure vehicle routes* performed by a complete vehicle serving only customers in N_v ; and finally *vehicle routes with subtours* performed by a complete vehicle. The latter type of route includes the case in which a trailer is detached at a vehicle customer in N_v to perform a subtour just with the truck visiting one or more customers in N_t (or even in N_v). The objective of the TTRP is to find a set of routes of minimum total distance such that: each customer is visited by a compatible vehicle exactly once; the total demand of the customers visited in a route or subtour does not exceed its capacity; and the number of required trucks and trailers is not greater than m_t and m_r , respectively.

The TTRP was introduced by Chao [4] and has been tackled using tabu search [4][9], simulated annealing [6], and a mathematical programming based heuristic [3]. Most of the methods ([3],[4],[9]) use a natural cluster-first, route-second approach. In this work, we show that a route-first, cluster-second procedure embedded within a hybrid metaheuristic based on Greedy Randomized

Adaptive Search Procedure (GRASP), Variable Neighbourhood Descent (VND) and Path Relinking (PR) is an effective approach to solve the TTRP. A description of the components of the hybrid metaheuristic follows.

2. Solution approach

GRASP is a two-phase iterative method: first, a feasible solution is built by a greedy randomized heuristic; second, the solution is improved by local search. As highlighted in [8], the performance of GRASP can be enhanced using multiple neighbourhoods and path relinking. Accordingly, we replaced the local search by an iterated VND, and used PR in various strategies.

Greedy Randomized Construction: originally proposed by Beasley [1] route-first, cluster-second methods provide a flexible and effective framework for the solution of arc and node routing problems [7]. Then, the greedy randomized construction of the proposed solution approach is performed by such a method. A giant tour $T = (0, t_1, \ldots, t_k, \ldots t_n)$ that visits all the customers in N is found using a randomized nearest neighbour heuristic with a restricted candidate list of size r. Then, a solution S of the TTRP is derived from T by means of a tour splitting procedure. The tour splitting procedure constructs one auxiliary acyclic graph $H = (X, U, W)$, where the set of nodes X contains a dummy node 0 and n nodes numbered 1 through n, and node k represents the customer in the $k-th$ position of T (i.e., t_k). The arc set U contains one arc $(k-1, l)$ if and only if the subsequence ($t_k, \ldots t_l$) can be served by a feasible route. Finally, the weight of the arc $(k-1, l)$ in W is the total distance of the corresponding route. To derive S it is necessary to find the shortest path between 0 and n in H. The cost of the shortest path corresponds to the total distance of S and the arcs in the shortest path represent the routes of S.

To adapt the tour splitting procedure for the solution of the TTRP it is necessary to take into account the heterogeneous fixed fleet. Thus, to obtain S, a resource-constrained shortest path problem is solved, where the resources are the available trucks and trailers. Moreover, if the arc $(k-1, l)$ represents a vehicle route with subtours its cost is found with a dynamic programming method that solves a restricted version of the Single Truck and Trailer Routing Problem with Satellite Depots [10]. Some preliminary experiments have shown that it may be difficult to find feasible solutions with the tour splitting procedure in problems with a tight ratio between the total demand and the total capacity. Therefore, if the solution of the resource-constrained shortest path problem fails to find a feasible solution, an unfeasible "solution" is obtained solving an unrestricted shortest path problem.

Iterated Variable Neighbourhood Descent: The improvement phase of the proposed method is performed with an iterated VND [5]. One main loop of the iterated VND takes S as initial solution and performs three steps: (1) randomly exchange p pairs of customers from its giant tour T to obtain a new giant tour T'; (2) derive a new solution S' by applying the tour splitting procedure to T'; and (3) apply VND to S'. The latter VND step uses five neighbourhoods in the following order: Or-opt (in single routes and subtours), node exchange, 2-opt, node relocation (in single routes/subtours and between pairs of routes/subtours), and finally, for each subtour it applies the root refining procedure of [4]. The exploration of each neighbourhood uses a best-improvement strategy, the iterated VND procedure repeats during ni iterations, and the value of p is controlled dynamically between 1 and $p_{\max}$. Since unfeasible solutions are accepted as initial solutions and also during the search of VND, the incumbent solution of VND is replaced by S' if $f(S') < f(S)$ and its unfeasibility $\mu(S') = \max\left\{0, \frac{nt(S')}{m_t} - 1\right\} + \max\left\{0, \frac{nr(S')}{m_r} - 1\right\}$ does not exceed a given limit τ, where $f(\cdot)$ denotes the objective function, and $nt(S')$ and $nr(S')$ the number of trucks and trailers used in S'. Every time a feasible solution is found, the best solution of the iterated VND is checked for an update. At each call of the iterated VND τ is initialized at $\tau_{\max}$, and updated at each iteration with $\tau = \tau - \tau_{\max}/ni$.

Path Relinking: GRASP with PR maintains a pool of elite solutions (ES). To be included in ES a solution S must be better than the worst solution of the pool; but to preserve its diversity, the distance between S and the pool ($d(ES, S)$) must be greater than a given threshold δ, where $d(ES, S) = \min_{S' \in ES} d(S', S)$, unless it is simply better than the best solution of ES. In this work the distance between any two solutions $d(S, S')$ is the distance for R-permutations [1] between their corresponding giant tours T and T'. The solutions in the pool are ordered according to a lexicographic comparator that gives priority to feasible solutions, among feasible solutions to those with smaller distances, and among unfeasible solutions to those with smaller unfeasibility. To transform the starting solution S_0 into the target solution S_f, the PR operator works in their giant tours, repairing from left to right the broken pairs of T_0 to create a path of giant tours with non-increasing distance to T_f. All the giant tours in the path are split and the resulting solutions are improved with VND, finally all the resulting solutions are tested for insertion in the pool. The PR operator uses the back and forward scheme [8], exploring the path from S_0 to S_f, and also the path from S_f to S_0. Due to the fact that GRASP and PR can be hybridized in different ways (see [8]), we tested PR: (i) as a post-optimization procedure; (ii) as an intensification mechanism; and (iii) in Evolutionary Path Relinking (EvPR).

3. Computational Results

The proposed method has been implemented in Java. All the variants were run for 60 GRASP iterations with $r = 2$, $ni = 200$, $p_{\max} = 6$, and $\tau_{\max} = 0.75$, for PR $|ES| = 5$ and $\delta = \max(10, m_t + m_r)$, and EvPR is run every 20 GRASP iterations. Table 1 shows the average results over the 21 instances described in [4]. In summary, all GRASP/VND with PR variants outperform the previous competing methods.

Table 1. Results for the 21 test instances of the TTRP. (BKS: Best known solution)

Method	Avg. Dev. BKS	Avg. Time (min)	Computer
GRASP/VND with EvPR	0.72%	46.39	Pentium D 3.4 GHz
GRASP/VND with PR (Post-optimization)	0.95%	28.77	Pentium D 3.4 GHz
GRASP/VND with PR (Intensification)	0.98%	37.33	Pentium D 3.4 GHz
Simulated annealing [6]	1.47%	39.56	Pentium IV 1.5 GHz
Math. Programming Heuristic [3]	1.70%	Not Available	Pentium IV 2.8 GHz
Tabu search [9]	1.71%	47.32	Pentium IV 1.5 GHz
Tabu search [4]	7.51%	14.51	Pentium II 350 MHz

References

[1] J.E. Beasley, “Route-first cluster-second methods for vehicle routing”. Omega 11, 403–408, 1983.

[2] V. Campos, M. Laguna and R. Martí, “Context-Independent Scatter and Tabu Search for Permutation Problems”, *INFORMS Journal on Computing* 17, 111-122 (2005).

[3] M. Caramia and F. Guerriero, “A heuristic approach for the truck and trailer routing problem”. *Journal of the Operational Research Society*. Advance online publication; doi:10.1057/jors.2009.59 (2009).

[4] I.-M. Chao, “A tabu search method for the truck and trailer routing problem”, *Computers & Operations Research* 29, 33-51 (2002).

[5] P. Hansen and N. Mladenovic, “Variable neighbourhood search: Principles and applications”, *European Journal of Operational Research* 130, 449-467 (2001).

[6] S.-W. Lin, V. F. Yu, and S.-Y. Chou, “Solving the truck and trailer routing problem based on a simulated annealing heuristic”, *Computers & Operations Research* 36, 1683-1692 (2009).

[7] C. Prins, N. Labadi, and M. Reghioui, “Tour splitting algorithms for vehicle routing problems”, *International Journal of Production Research* 47, 507-535(2009).

[8] M.G.C. Resende and C.C. Ribeiro, “Greedy Randomized Adaptive Search Procedures: advances and applications”, in *Handbook of Metaheuristics, 2nd Ed.* M. Gendreau and J.-Y. Potvin (eds.), Springer, (To appear).

[9] S. Scheuerer, “A tabu search heuristic for the truck and trailer routing problem”, *Computers & Operations Research* 33, 894-909 (2006).

[10] J.G. Villegas, C. Prins, C. Prodhon, A.L. Medaglia and N. Velasco. GRASP/VND and multi-start evolutionary local search for the single truck and trailer routing problem with satellite depots. *Engineering Applications of Artificial Intelligence,* Forthcoming (2010).